



Correction

Exercice 1

1. a. 
$$\det M = \begin{vmatrix} 1 & 1 & -4 \\ -2 & -3 & 8 \\ 3 & 4 & -11 \end{vmatrix} = \begin{vmatrix} 1 & 1 & -4 \\ 0 & -1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = 1 \times \begin{vmatrix} -1 & 0 \\ 1 & 1 \end{vmatrix} = -1.$$

$\det M \neq 0$  donc  $M$  est inversible.

b. 
$$\begin{pmatrix} 1 & 1 & -4 & \vdots & 1 & 0 & 0 \\ -2 & -3 & 8 & \vdots & 0 & 1 & 0 \\ 3 & 4 & -11 & \vdots & 0 & 0 & 1 \end{pmatrix} \begin{array}{l} L_2 \leftarrow L_2 + 2L_1 \\ L_3 \leftarrow L_3 - 3L_1 \end{array}$$

$$\begin{pmatrix} 1 & 1 & -4 & \vdots & 1 & 0 & 0 \\ 0 & -1 & 0 & \vdots & 2 & 1 & 0 \\ 0 & 1 & 1 & \vdots & -3 & 0 & 1 \end{pmatrix} \begin{array}{l} L_3 \leftarrow L_3 + L_2 \\ L_2 \leftarrow -3L_2 \end{array}$$

$$\begin{pmatrix} 1 & 1 & -4 & \vdots & 1 & 0 & 0 \\ 0 & 1 & 0 & \vdots & -2 & -1 & 0 \\ 0 & 0 & 1 & \vdots & -1 & 1 & 1 \end{pmatrix} \begin{array}{l} L_1 \leftarrow L_1 - L_2 + 4L_3 \end{array}$$

$$\begin{pmatrix} 1 & 0 & 0 & \vdots & -1 & 5 & 4 \\ 0 & 1 & 0 & \vdots & -2 & -1 & 0 \\ 0 & 0 & 1 & \vdots & -1 & 1 & 1 \end{pmatrix}$$

Donc  $M^{-1} = \begin{pmatrix} -1 & 5 & 4 \\ -2 & -1 & 0 \\ -1 & 1 & 1 \end{pmatrix}.$

2. Le système est équivalent à  $MX = B$  avec  $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$  et  $B = \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix}.$

Donc  $X = M^{-1} \times B = \begin{pmatrix} -1 & 5 & 4 \\ -2 & -1 & 0 \\ -1 & 1 & 1 \end{pmatrix} \times \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}.$

D'où  $\begin{cases} x = 1 \\ y = 5 \\ z = 2 \end{cases}.$

Et  $S = \{(1;5;2)\}.$

## Exercice 2

a. On pose  $u(x) = \ln x$   $v'(x) = x^2$ .  
 D'où  $u'(x) = \frac{1}{x}$   $v(x) = \frac{1}{3}x^3$ .  

$$\int x^2 \ln x \, dx = \frac{1}{3}x^3 \ln x - \int \frac{1}{x} \times \frac{1}{3}x^3 \, dx = \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^2 \, dx = \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 = \frac{1}{9}x^3(3 \ln x - 1).$$

b. On a  $\int \sqrt{1+3\cos^2 x} \sin 2x \, dx = \int 2\sqrt{1+3\cos^2 x} \sin x \cos x \, dx$ .  
 Or  $(1+3\cos^2 x)' = -6\sin x \cos x$ .  
 Donc  $\int \sqrt{1+3\cos^2 x} \sin 2x \, dx = -\frac{2}{3} \times \frac{1}{3} \times (1+3\cos^2 x)^{3/2} = -\frac{2}{9}(1+3\cos^2 x)^{3/2}$ .

c.  $x^2+x+1=0$   $\Delta = 1^2 - 4 \times 1 \times 1 = 1 - 4 = -3 < 0$   

$$\frac{x^2-x+3}{x^2+x+1} = \frac{x^2+x+1-2x+2}{x^2+x+1} = 1 - \frac{2x-2}{x^2+x+1} = 1 - \frac{2x+1}{x^2+x+1} + \frac{3}{x^2+x+1}$$
  
 Or  $x^2+x+1 = \left(x+\frac{1}{2}\right)^2 + \frac{3}{4}$ .  
 D'où  $\int \frac{x^2-x+3}{x^2+x+1} \, dx = \int 1 - \frac{2x+1}{x^2+x+1} \, dx + \int \frac{3}{\left(x+\frac{1}{2}\right)^2 + \frac{3}{4}} \, dx$   

$$= x - \ln|x^2+x+1| + \int \frac{3}{\left(x+\frac{1}{2}\right)^2 + \frac{3}{4}} \, dx$$

On pose  $u = \frac{2\sqrt{3}}{3}\left(x+\frac{1}{2}\right)$ .

On a  $x = \frac{\sqrt{3}}{2}u - \frac{1}{2}$  et  $dx = \frac{\sqrt{3}}{2}du$ .

D'où  $\int \frac{x^2-x+3}{x^2+x+1} \, dx = x - \ln(x^2+x+1) + \int \frac{3}{\frac{4}{3}u^2 + \frac{3}{4}} \times \frac{\sqrt{3}}{2} \, du$   

$$= x - \ln(x^2+x+1) + \frac{4}{3} \times 3 \times \frac{\sqrt{3}}{2} \int \frac{1}{u^2+1} \, du$$
  

$$= x - \ln(x^2+x+1) + 2\sqrt{3} \arctan u$$
  

$$= x - \ln(x^2+x+1) + 2\sqrt{3} \arctan \frac{2\sqrt{3}}{3}\left(x+\frac{1}{2}\right).$$

## Exercice 3

$\cos x = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + o(x^4)$ .

$\operatorname{ch} x = 1 + \frac{1}{2}x^2 + \frac{1}{24}x^4 + o(x^4)$ .

$\cos x \operatorname{ch} x = \left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4\right) \times \left(1 + \frac{1}{2}x^2 + \frac{1}{24}x^4\right) + o(x^4)$   

$$= 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + \frac{1}{2}x^2 - \frac{1}{4}x^4 + \frac{1}{24}x^4 + o(x^4)$$
  

$$= 1 - \frac{1}{6}x^4 + o(x^4).$$

$1$	$-\frac{1}{6}x^4$	$3+x^2$
$1 + \frac{1}{3}x^2$		$\frac{1}{3} - \frac{1}{9}x^2 - \frac{1}{54}x^4$
$-\frac{1}{3}x^2 - \frac{1}{6}x^4$		
$-\frac{1}{3}x^2 - \frac{1}{9}x^4$		
$-\frac{3}{54}x^4$		

D'où  $\frac{\cos x \operatorname{ch} x}{3+x^2} = \frac{1}{3} - \frac{1}{9}x^2 - \frac{1}{54}x^4 + o(x^4)$ .

#### Exercice 4

On a  $\arctan x = x + o(x^2)$ , donc,  $1 + x \arctan x = 1 + x^2 + o(x^3)$ .

De plus,  $\cos x = 1 - \frac{1}{2}x^2 + o(x^3)$ .

D'où  $1 + x \arctan x - \cos x = \frac{3}{2}x^2 + o(x^2)$ .

On obtient alors  $1 + x \arctan x - \cos x \sim \frac{3}{2}x^2$  et  $\lim_{x \rightarrow 0} \frac{1 + x \arctan x - \cos x}{x^2} = \frac{3}{2}$ .

#### Exercice 5

$$J_f = \begin{pmatrix} y & x & 0 \\ -z \sin x \cos y & -z \cos x \sin y & \cos x \cos y \\ z \cos x \sin y & z \sin x \cos y & \sin x \sin y \end{pmatrix}.$$

$$\begin{aligned} |J_f| &= \begin{vmatrix} y & x & 0 \\ -z \sin x \cos y & -z \cos x \sin y & \cos x \cos y \\ z \cos x \sin y & z \sin x \cos y & \sin x \sin y \end{vmatrix} \\ &= y \times \begin{vmatrix} -z \cos x \sin y & \cos x \cos y \\ z \sin x \cos y & \sin x \sin y \end{vmatrix} - x \times \begin{vmatrix} -z \sin x \cos y & \cos x \cos y \\ z \cos x \sin y & \sin x \sin y \end{vmatrix} \\ &= yz \times \begin{vmatrix} -\cos x \sin y & \cos x \cos y \\ \sin x \cos y & \sin x \sin y \end{vmatrix} - xz \times \begin{vmatrix} -\sin x \cos y & \cos x \cos y \\ \cos x \sin y & \sin x \sin y \end{vmatrix} \\ &= yz \cos x \sin x \times \begin{vmatrix} -\sin y & \cos y \\ \cos y & \sin y \end{vmatrix} - xz \cos y \sin y \times \begin{vmatrix} -\sin x & \cos x \\ \cos x & \sin x \end{vmatrix} \\ &= yz \times (-\cos x \sin x) - xz \times (-\cos y \sin y) \\ &= xz \cos y \sin y - yz \cos x \sin x \\ &= \frac{z}{2}(x \sin 2y - y \sin 2x). \end{aligned}$$