

Exploiting Historical Relationships of Clauses and Variables in Local Search for Satisfiability (Poster Presentation)

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Variable properties such as score and age are used to select a variable to flip. The score of a variable x refers to the decrease in the number of unsatisfied clauses if x is flipped. The age of x refers to the number of steps done since the last time when x was flipped. If the best variable according to scores in a randomly chosen unsatisfied clause c is not the youngest in c , *Novelty* [4] flips this variable. Otherwise, with probability p (noise p), *Novelty* flips the second best variable, and with probability $1-p$, *Novelty* flips the best variable. *Novelty+* [1] randomly flips a variable in c with probability wp and does as *Novelty* with probability $1-wp$. *Novelty++* [3] flips the least recently flipped variable (oldest) in c with probability dp , and does as *Novelty* with probability $1-dp$.

The above approaches just use the current properties of variables to select a variable to flip. These approaches are effective for the problems that do not present uneven distribution of variable and/or clause weights during the search. In other words, problems can be solved using these approaches when there are no clauses or variables whose weight is several times larger than the average during the search [6]. However, when solving hard random or structured SAT problems using these approaches, variable or clause weight distribution is often uneven. In this case, the falsification history of clauses and/or the flipping history of variables during the search should be exploited to select a variable to flip for the problems to be solved.

In this paper, we present a noise mechanism that exploits the history information to determine noise p . For a falsified clause c , let $var_fals[c]$ denote the variable that most recently falsifies c and let $num_fals[c]$ denote the number of the most recent consecutive falsifications of c due to the flipping of this variable. If the best variable in c is not $var_fals[c]$, this variable is flipped. Otherwise, the second best variable is flipped with probability p , where p is determined as a function of $k=num_fals[c]$: $\{20, 50, 65, 72, 78, 86, 90, 95, 98, 100\}$, i.e., $p=0.2$ if $k=1$, $p=0.5$ if $k=2$, ..., $p=1$ if $k \geq 10$. This probability vector was empirically turned using a subset of instances.

Another adaptive noise mechanism was introduced in [2] to automatically adjust noise during the search. We refer to this mechanism as Hoos's noise mechanism.

SLS solvers *TNM* and *adaptG²WSAT2011*, described in Fig. 1, are both based on *Novelty*, but use noise p determined by our noise mechanism at each **uneven** step, and use noise $p1$ adjusted by Hoos noise mechanism at each **even** step, to flip the second best variable in the randomly selected unsatisfied clause c when the best variable in c is the youngest in c . In *TNM*, a step is even if the variable weight distribution currently is even (e.g., all variable weights are smaller than $10 \times$ the average variable weight). In *adaptG²WSAT2011*, a step is even if c currently has small weight (e.g., smaller than $10 \times$ the average clause weight). Otherwise the step is uneven.

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1:  $A \leftarrow$  randomly generated truth assignment;
2: for  $flip \leftarrow 1$  to  $Maxsteps$  do
3:   if  $A$  satisfies SAT-formula  $\mathcal{F}$  then return  $A$ ;
4:   adjust noise  $p1$  according to Hoos's noise mechanism;
5:   if there is any promising decreasing variable
6:   then  $y \leftarrow$  the promising decreasing variable with the largest score;
7:   else randomly select an unsatisfied clause  $c$ ;
8:     if the current search step is even
9:     then
10:       $y \leftarrow$  heuristic  $Novelty+$  applied to  $c$  using probability  $p1$  (and  $wp=p1/10$ );
11:     else
12:      determine noise  $p$  according to  $var\_fals[c]$  and  $num\_fals[c]$ ;
13:       $y \leftarrow$  heuristic  $Novelty++$  applied to  $c$  using probability  $p$  (and  $dp=p/10$ );
14:    $A \leftarrow A$  with  $y$  flipped;
15: return Solution not found;

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Fig. 1. Algorithms TNM and $adaptG^2$ $WSAT2011$

Our noise mechanism is different from Hoos's noise mechanism in two respects. First, our mechanism uses the history of most recent consecutive falsifications of a clause due to the flipping of one variable, while Hoos's noise mechanism observes the improvement in the objective function. Second, the noise determined by our mechanism is clause-specific, while the noise adjusted by Hoos's noise mechanism is not.

$Novelty$, $Novelty+$, or $Novelty++$ considers the last falsification of c . TNM and $adaptG^2$ $WSAT2011$ extend them by considering more historical falsifications of c .

TNM won a gold medal in the 2009 SAT competition in the random category and is competitive for crafted instances. In the 2011 SAT competition, $ppfolio$ won two gold medals in the crafted sat category, because TNM solved 75 instances, while its other four constituent algorithms solved 24, 18, 7, and 3 instances, respectively [5]. Solver $adaptG^2$ $WSAT2011$ performs better than TNM . In the 2011 SAT competition, it solved 7 more instances than TNM in the random category in phase 1. In addition, $adaptG^2$ $WSAT2011$ solved 99 instances in the crafted category in phase 1, while the best CDCL solver solved 81 instances in phase 1 (and 93 instances in phase 2).

References

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