Robust estimation of vehicle lateral velocity and yaw rate using Switched T-S Fuzzy Interval Observer

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Outline of the talk

- Context: estimation of vehicle lateral dynamics
- Problem Statement and Some Background
- System description
- Switched Interval Observer Design
- Experimental validation
- Conclusion
Knowledge of vehicle lateral velocity is essential for safety enhancement, in particular for development of active safety systems.
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As a consequence the virtual sensing approach (observer) proposed here may be of particular interest.
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As a consequence the virtual sensing approach (observer) proposed here may be of particular interest.

Goal: Robust estimation process of vehicle lateral velocity and yaw rate taking into account: Model uncertainty and changes in operating conditions.
Problem statement

Reduces tires at each axle to a single equivalent tire;

The vehicle state is described by body-fixed lateral velocity and yaw rate.

Figure: Bicycle Model.
The dynamics equations can be represented by (Rajamani(2011)):

\[
\begin{align*}
    m\ddot{v}_y + mr &= F_{yf} + F_{yr} \\
    l_z \dot{r} &= l_f F_{yf} - l_r F_{yr}
\end{align*}
\] (1)

where $m, I_z, l_r, l_f$ denote respectively the mass of the vehicle, the yaw moment and the distances from the rear and the front axle to the center of gravity.

$v_x$ is a time-varying longitudinal velocity, $v_y$ is the lateral velocity of the vehicle and $r$ is the yaw rate. $F_{yr}$ and $F_{yf}$ are the lateral rear and front forces respectively.
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- $v_x$ is a time-varying longitudinal velocity, $v_y$ is the lateral velocity of the vehicle and $r$ is the yaw rate.
- $F_{yr}$ and $F_{yf}$ are the lateral rear and front forces respectively.
Using Pacejka’s magic formula (Pacejka and Bakker (1991)), the lateral forces are given by:

\[ F_{yi} = D_i \sin(C_i \tan^{-1}(B_i(1 - E_i)\alpha_i + E_i \tan^{-1}(B_i \alpha_i))) \]  

(2)

- \( i = \{r, f\} \) denotes rear and front of the vehicle;
- \( D_i, C_i, B_i \) and \( E_i \) are the characteristic constants of the tires.
- \( \alpha_f \) and \( \alpha_r \) are respectively the front and rear sideslip angles of the tires.
Problem Statement

- Pacejka's magic formula:
  Nonlinear model;
Problem Statement

- Pacejka's magic formula: Nonlinear model;

\[ F_y = c_i \alpha \]

Where:
- \( F_y \) is the lateral force applied to the tire.
- \( c_i \) denotes the cornering stiffness of tires.
- \( \alpha \) is the sideslip angle.

The graph shows the relationship between lateral force \( F_y \) and sideslip angle \( \alpha \). The curve peaks at a certain angle, indicating the nonlinear behavior of the tire at different road conditions.
Problem Statement

- Pacejka's magic formula: Nonlinear model;

- Nominal conditions & small sideslip angles: \( c_i \) fixed

\[
F_{yi} = c_i \alpha_i
\]

\( c_i \) denotes the cornering stiffness of tires.
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- Change on road conditions or nonlinear region is reached: $c_i$ variable
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$$F_{yi} = c_i \alpha_i$$

$c_i$ denotes the cornering stiffness of tires.

- Change on road conditions or nonlinear region is reached: $c_i$ variable

- In practice, the cornering stiffness coefficients are not constant but time varying.
Proposed Approach

- Existing approaches → Cornering stiffness parameters are constants
  \[ F_{yi} = c_i \alpha_i \]

- Proposed approach → Cornering stiffness parameters are uncertain
  \[ F_{yi} = (c_{i0} + \Delta c_i) \alpha_i \]

- Assumption:
  \[ \Delta c_i^- \leq \Delta c_i \leq \Delta c_i^+ \]
Proposed Approach

- **Interval Observers:** Under assumptions of knowing bounds on uncertain terms and initial conditions → Estimation of a feasible solution set of vehicle lateral velocity and yaw rate;

\[ x^+(t), x(t), x^-(t) \]

**Main contribution.** A new estimation process for vehicle’s lateral velocity and yaw rate presenting many benefits over the existing state of art works, within the dynamic estimation framework.
Proposed Approach

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- **Main contribution.** A new estimation process for vehicle’s lateral velocity and yaw rate presenting many benefits over the existing state of art works, within the dynamic estimation framework.
Vehicle Lateral Dynamic model:

\[
\begin{bmatrix}
\dot{v}_y \\
\dot{\gamma}
\end{bmatrix} = \begin{bmatrix}
-\frac{c_f + c_r}{mv_x} & \frac{c_r l_r - c_f l_f}{mv_x} \\
\frac{c_r l_r - c_f l_f}{l_z v_x} & \frac{c_r l_r^2 + c_f l_f^2}{l_z v_x}
\end{bmatrix} \begin{bmatrix}
v_y \\
r
\end{bmatrix} + \begin{bmatrix}
\frac{c_f}{m} \\
\frac{c_f}{l_z}
\end{bmatrix} \delta_f
\]

(3)

where longitudinal velocity and cornering stiffness are treated respectively as the \textit{measurable} and \textit{unmeasurable} time varying parameters.
System Description

- Vehicle Lateral Dynamic model:

\[
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\dot{r}
\end{bmatrix} = 
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- \frac{c_r + c_f}{mv_x} & \frac{c_r l_r - c_f l_f}{mv_x} \\
\frac{c_r l_r - c_f l_f}{l_z v_x} & - \frac{c_r l_r^2 + c_f l_f^2}{l_z v_x}
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\end{bmatrix} \delta_f
\] (3)

where longitudinal velocity and cornering stiffness are treated respectively as the measurable and unmeasurable time varying parameters.

- LPV state-space model

\[
\begin{align*}
\dot{x}(t) &= A(\rho(t), \xi(t))x(t) + B(\xi(t))u(t) \\
y(t) &= Cx(t)
\end{align*}
\] (4)

where \( \rho(t) = \left[ \frac{1}{v_x} \ v_x \right]^T \) and \( \xi(t) = \left[ c_r \ c_f \right]^T \).
Adopting a switching strategy based on longitudinal velocity variation range, a switched linear parameter-varying model for the vehicle lateral dynamics is derived

\[
\begin{cases}
\dot{x}(t) = A_{\sigma(t)}(\rho(t), \xi(t))x(t) + B(\xi(t))u(t) \\
y(t) = Cx(t)
\end{cases}
\]

\(\sigma(t): \mathbb{R}_+ \rightarrow \mathcal{I} : \{1, \ldots, N\}\) is a Switching law that indicates at each time which mode is active.
System Description

- Adopting a **switching strategy** based on longitudinal velocity variation range, a switched linear parameter-varying model for the vehicle lateral dynamics is derived

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\]

(5)

\(\sigma(t) : \mathbb{R}_+ \rightarrow \mathcal{I} : \{1, \ldots, N\}\) is a Switching law that indicates at each time which mode is active.

- **Takagi-Sugeno (T-S) switched system**

\[
\begin{align*}
\dot{x}(t) &= \sum_{j=1}^{4} h_{\sigma(t)}^j(\rho(t))A_{\sigma(t)}^j(\xi(t))x(t) + B(\xi(t))u(t) \\
y(t) &= Cx(t)
\end{align*}
\]

(6)

where \(\rho(t)\) is the decision variable and \(h_{\sigma(t)}^j(\rho(t))\) are switched weighting functions, \(\forall j \in \{1, \ldots, 4\}\).

- The activating functions \(h_{\sigma(t)}^j(\rho(t))\) satisfy the convex sum properties

\[
\sum_{j=1}^{4} h_{\sigma(t)}^j(\rho(t)) = 1, \quad 0 \leq h_{\sigma(t)}^j(\rho(t)) \leq 1
\]

(7)
Background on Interval observer design

Before state the main results...

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- For any two vectors $x_1$, $x_2$ or matrices $M_1$, $M_2$ the inequalities $x_1 \leq x_2$, $x_1 \geq x_2$, $M_1 \leq M_2$ and $M_1 \geq M_2$ must be interpreted element-wise.

- A real matrix $A_i$, $\forall i \in \mathcal{I}$ is called a Metzler matrix if all its elements outside the main diagonal are positive, i.e,

\[
\exists \beta \geq 0, \ A_i + \beta I_n \geq 0 \tag{8}
\]

---

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\]  
(8)

An important application of **positive dynamics**

**Lemma 1. Positive Switched Systems**

For a Metzler matrix $A_i$, $\forall i \in \mathcal{I}$, the switched system

\[
\dot{x}(t) = A_{\sigma(t)}x(t) + \delta_{\sigma(t)}(t)
\]  
(9)

is said to be a positive switched system \(^1\) if $x(t_0) \geq 0$, $A_i$ is a $n \times n$ Metzler matrix and $\delta_i(t) \geq 0 \ \forall i \in \{1, ..., N\}$.

Switched Interval Observer Design

Assumptions

- **Assumption 1.** The pair \((A_{i}^{j+}, C)\) is detectable \(\forall i \in I, j \in \{1, \ldots, 4\}, t \geq 0\).

- **Assumption 2.** There exist known functions \(u^{-}(t), u^{+}(t) \in \mathbb{R}^{m}\) such that

\[
u^{-}(t) \leq u(t) \leq u^{+}(t), \quad \forall t \geq t_{0} \tag{10} \]

- **Assumption 3.** There exist known constants matrices \(A_{i}^{j+}, A_{i}^{j-}, B^{+}, B^{-} \forall i \in I, j \in \{1, \ldots, 4\}, \forall \rho(t) \in \nabla_{i}\) and \(\forall \xi(t) \in \Xi = \begin{bmatrix} [c_{r}^{-}, c_{r}^{+}] & [c_{f}^{-}, c_{f}^{+}] \end{bmatrix}^{T}\) such that:

\[
A_{i}^{j-} \leq A_{i}^{j}(\xi(t)) \leq A_{i}^{j+} \\
B^{-} \leq B(\xi(t)) \leq B^{+}
\]

The matrices \(A_{i}^{j-}, A_{i}^{j+}, B^{+}\) and \(B^{-}\) can be directly calculated using the known subset \(\Xi\).

- **Notations.** \(A_{\sigma(t)}(\rho(t), \xi_{0}) \mapsto A_{\sigma(t), \rho, \xi_{0}}\).
Theorem 1.

Assuming that the trajectory of system (6) is bounded $\|x\| \leq x$, $\forall t \geq t_0$. Then, for all initial conditions $x_0$ such that $x^-_0 \leq x_0 \leq x^+_0$, there exists a convergent switched interval observer of the TS model (6) of the form:

$$
\begin{align*}
\dot{x}^+(t) &= \sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t))(A^+_{j\sigma(t)}x^+(t) + L^j_{\sigma(t)}(y - Cx^+(t)) + B^+ u^+(t) + \\
(A^+_{j\sigma(t)} - A_{\sigma(t),\rho,\xi_0})(|x^+(t)| - x^+(t))) \\
\dot{x}^-(t) &= \sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t))(A^-_{j\sigma(t)}x^-(t) + L^j_{\sigma(t)}(y - Cx^-(t)) + B^- u^-(t) - \\
(A^-_{j\sigma(t)} - A_{\sigma(t),\rho,\xi_0})(|x^-(t)| + x^-(t)))
\end{align*}
$$

(11)

if the matrix $\sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t))(A^-_{j\sigma(t)} - L^j_{\sigma(t)} C)$ is Metzler and the matrix $\sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t))(A^+_{j\sigma(t)} - L^j_{\sigma(t)} C)$ is Hurwitz $\forall \rho(t) \in \nabla_{\sigma(t)}$ and $\forall \xi(t) \in \Xi$. 


Sufficient conditions for boundedness
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The upper estimation error $e^{+}(t) = x^{+}(t) - x(t)$ is governed by the following equation

$$
\dot{e}^{+}(t) = \sum_{j=1}^{4} h^{j}_{\sigma(t)}(\rho(t))((A_{\sigma(t), \rho, \xi_0} - L^{j}_{\sigma(t)}C)e^{+}(t) + \delta^{j+}_{\sigma(t)}(t)
$$

(12)
① **Sufficient conditions for boundedness**

The upper estimation error $e^+(t) = x^+(t) - x(t)$ is governed by the following equation

$$
\dot{e}^+(t) = \sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t))((A_{\sigma(t)},\rho,\xi_0 - L^j_{\sigma(t)}C)e^+(t) + \delta^j_{\sigma(t)}(t) \quad (12)
$$

where by construction $\delta^j_{\sigma(t)}(t) \geq 0$. 


Switched Interval Observer Design Elements of Proof

1. Sufficient conditions for boundedness

The upper estimation error $e^+(t) = x^+(t) - x(t)$ is governed by the following equation

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\dot{e}^+(t) = \sum_{j=1}^{4} h_{\sigma(t)}^j(\rho(t))((A_{\sigma(t)},\rho,\xi_0 - L_{\sigma(t)}^j)C)e^+(t) + \delta_{\sigma(t)}^{i+}(t)
$$

(12)

where by construction $\delta_{\sigma(t)}^{i+}(t) \geq 0$.

Remark 1. It's clear that if $\sum_{j=1}^{4} h_{\sigma(t)}^j(\rho(t))(A_{\sigma(t)}^{i-} - L_{\sigma(t)}^j)C$ is Metzler then

$$
\sum_{j=1}^{4} h_{\sigma(t)}^j(\rho(t))((A_{\sigma(t)},\rho,\xi_0 - L_{\sigma(t)}^j)C)\text{ is also Metzler for any } A_{\sigma(t),\rho,\xi_0} \text{ in the interval:}
$$

$$A_i^{i-} \leq A_i,\rho,\xi_0 \leq A_i^{i+} \quad \forall i \in \mathcal{I}, \forall j \in \{1, 2, 3, 4\}$$
Switched Interval Observer Design

Elements of Proof

① Sufficient conditions for boundedness

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\dot{e}^+(t) = \sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t))((A_{\sigma(t)},\rho,\xi_0 - L^j_{\sigma(t)}C)e^+(t) + \delta^j_{\sigma(t)}(t) \tag{12}
\]

where by construction \( \delta^j_{\sigma(t)}(t) \geq 0 \).

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\sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t))((A_{\sigma(t)},\rho,\xi_0 - L^j_{\sigma(t)}C) \] is also Metzler for any \( A_{\sigma(t)},\rho,\xi_0 \) in the interval:

\[
A^j_{i^-} \leq A_{i,\rho,\xi_0} \leq A^j_{i^+} \quad \forall i \in \mathcal{I}, \forall j \in \{1, 2, 3, 4\}
\]

Under Lemma 1, if \( \sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t))(A^j_{\sigma(t)} - L^j_{\sigma(t)}C) \) is a Metzler Matrix, then the dynamics of \( e^+(t) \) is positive, it follows that
\textbf{Switched Interval Observer Design} \hfill \textbf{Elements of Proof}

\begin{itemize}
\item \textit{Sufficient conditions for boundedness}
\end{itemize}

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$$

(12)

where by construction $\delta^j_{\sigma(t)}(t) \geq 0$.

\textbf{Remark 1.} It's clear that if $\sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t))(A^j_{\sigma(t)} - L^j_{\sigma(t)})$ is Metzler then $\sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t))((A_{\sigma(t)},\rho,\xi_0 - L^j_{\sigma(t)})C)$ is also Metzler for any $A_{\sigma(t)},\rho,\xi_0$ in the interval:

$$
A^j_{\sigma(t)} \leq A_{\sigma(t)},\rho,\xi_0 \leq A^j_{\sigma(t)} \quad \forall i \in \mathcal{I}, \forall j \in \{1, 2, 3, 4\}
$$

Under Lemma 1, if $\sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t))(A^j_{\sigma(t)} - L^j_{\sigma(t)})$ is a Metzler Matrix, then the dynamics of $e^+(t)$ is positive, it follows that $e^+(t) \geq 0 \Rightarrow x(t) \leq x^+(t)$. 
By the same reasoning, it follows that if \( \sum_{j=1}^{4} h_{\sigma(t)}^j(\rho(t))(A_{\sigma(t)}^j - L_{\sigma(t)}^j C) \) is Metzler, then the lower estimation error \( e^-(t) = x(t) - x^- (t) \geq 0 \Rightarrow x^- (t) \leq x(t) \), implies that \( x^- (t) \leq x(t) \leq x^+(t) \)
By the same reasoning, it follows that if \[ \sum_{j=1}^{4} h_{\sigma(t)}^j(\rho(t))(A_{\sigma(t)}^{j-} - L_{\sigma(t)}^j C) \] is Metzler, then the lower estimation error \( e^{-}(t) = x(t) - x^{-}(t) \geq 0 \implies x^{-}(t) \leq x(t) \), implies that 
\[
x^{-}(t) \leq x(t) \leq x^{+}(t)
\]

**Problem 1.**

Find the gain matrix \( L_{\sigma(t)}^j \) such that \[ \sum_{j=1}^{4} h_{\sigma(t)}^j(\rho(t))(A_{\sigma(t)}^{j-} - L_{\sigma(t)}^j C) \] is Metzler \( \forall j \in \{1, \ldots, 4\}, \forall \sigma(t) \).
Sufficient conditions for convergence

The dynamics of the total error $e(t) = x^+(t) - x^-(t)$ is given by

$$\dot{e}(t) = \sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t)) \left( (A^{ij}_{\sigma(t)} - L^j_{\sigma(t)} C)e(t) + \delta^j_{\sigma(t)}(t) \right)$$

(13)

where by construction $\delta^j_{\sigma(t)}(t) \geq 0$. 
2 Sufficient conditions for convergence

The dynamics of the total error $e(t) = x^+(t) - x^-(t)$ is given by

$$\dot{e}(t) = \sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t)) \left( (A^+_{\sigma(t)} - L^j_{\sigma(t)}C)e(t) + \delta^j_{\sigma(t)}(t) \right)$$

(13)

where by construction $\delta^j_{\sigma(t)}(t) \geq 0$.

Problem 2.

Find the gain matrix $L^j_{\sigma(t)}$ such that $\sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t))(A^+_{\sigma(t)} - L^j_{\sigma(t)}C)$ is Input-to-State Stable with respect to $\delta^j_{\sigma(t)}(t)$. 
The closed-loop stability is studied using a **Switched Fuzzy ISS-Lyapunov Function**

\[
V(e(t)) = \sum_{i=1}^{N} \sum_{j=1}^{4} \lambda_i(t) h_i^j(\rho(t)) e^T(t) P_i^j e(t)
\]  

(14)

where \(\lambda(t)\) represent the indicator function specifying the current active subsystem and \(P_i^j\) represent the i-th diagonal positive matrix.

These properties are satisfied

\[
\lambda_i(t) \geq 0, \quad \forall i \in \mathcal{I}, \quad \sum_{i=1}^{N} \lambda_i(t) = 1, \quad \sum_{i=1}^{N} \dot{\lambda}_i(t) = 0
\]

\[
\sum_{i=1}^{N} \sum_{k=1}^{4} \lambda_i(t) \dot{h}_i^k(\rho(t)) = 0
\]
The closed-loop stability is studied using a **Switched Fuzzy ISS-Lyapunov Function**

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\[
\forall i \in I, \quad \sum_{i=1}^{N} \lambda_i(t) = 1, \quad \sum_{i=1}^{N} \dot{\lambda}_i(t) = 0
\]

\[
\sum_{i=1}^{N} \sum_{k=1}^{4} \lambda_i(t) \dot{h}_i^k(\rho(t)) = 0
\]

It can be shown that:

\[
\dot{V}_i(e(t)) < -\varepsilon V_i(e(t)) + \gamma \sum_{i=1}^{N} \sum_{j=1}^{4} \lambda_i(t) h_i^j(\rho(t)) \delta_i^T(t) \delta_i^j(t)
\]

(15)
The closed-loop stability is studied using a **Switched Fuzzy ISS-Lyapunov Function**

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These properties are satisfied

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\]

\[
\sum_{i=1}^{N} \sum_{j=1}^{4} \lambda_i(t) h_i^j(\rho(t)) = 0
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It can be shown that:

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\]  

(15)

Asymptotic stability is no longer ensured

Convergence in a ball around the origin, to be minimized using ISS property
Theorem 2.

Assuming that

\[ \sum_{i=1}^{N} \lambda_i(t) |h_i^k(\rho(t))| \leq \sum_{i=1}^{N} \lambda_i(t) \phi_i^k \]  

(16)

where \( \phi_i^k \geq 0 \) (\( k = 1, \ldots, 4 \)) are given scalars, if there exist, diagonal positive definite matrices \( P_i^j \), matrices \( W_i^j \) and \( M_i \), \( \forall i \in \mathcal{I}, j = \{1, \ldots, 4\} \), \( \gamma > 0 \) for given positive scalars \( \epsilon \) and \( \epsilon \) such that the following conditions hold

\[
\min_{P_i^j, M_i, W_i^j} \gamma \\

P_i^j \succ 0 \\
P_i^k + M_i \succ 0 
\]  

(17) \hspace{1cm} (18)
\[
\begin{bmatrix}
\Lambda^j_i + \epsilon P^j_i + \sum_{k=1}^{4} (\phi^k_i P^k_i + M_i) & P^j_i \\
\end{bmatrix} \prec 0 \quad (19)
\]

\[
P^j_i A^+_i - W^j_i C + \epsilon P^j_i \geq 0 \quad (20)
\]

where

\[
\Lambda^j_i = A^+_i P^j_i - C^T W^j_i P^j_i + P^j_i A^+_i - W^j_i C
\]

Then the proposed observer can estimate the lower and upper bounds of the state vector \( \hat{x}(t) \) for any switching signal, where \( L^j_i = P^j_i^{-1} W^j_i \).
Experimental validation

- The experimental data are acquired with a prototype vehicle;
- The run was performed on a test track located in the city of Versailles-Satory (France);
- The track is 3.5Km length with various curve profiles allowing vehicle dynamics excitation.

\(^2\)This measure is not used for observer design. It serves only for observer estimation evaluation.
Experimental validation

- The experimental data are acquired with a prototype vehicle;
- The run was performed on a test track located in the city of Versailles-Satory (France);
- The track is 3.5Km length with various curve profiles allowing vehicle dynamics excitation.

Several sensors are implemented on the vehicle:

1. An inertial unit provide the yaw rate $r$ measurement;
2. An absolute optical encoder to measure the steering angle $\delta_f$;
3. An odometer to measure the vehicle longitudinal speed $v_x$;
4. A high precision Correvit sensor provide a measure of the sideslip angle $^2$.

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$^2$This measure is not used for observer design. It serves only for observer estimation evaluation.
Experimental validation

- The longitudinal velocity should be treated as a time-varying parameter;

- The cornering stiffness parameters are affected by 10% uncertainty in their nominal value.

- In this scenario, the lateral forces reach the nonlinear zone.

**Figure:** Steering angle.

**Figure:** Longitudinal velocity.
Experimental validation

Consider the following switching law

\[ \sigma(t) = \begin{cases} 
1 & \text{if } 0 < v_x \leq 6 \text{m.s}^{-1} \\
2 & \text{if } 6 \text{m.s}^{-1} < v_x \leq 11 \text{m.s}^{-1} \\
3 & \text{if } 11 \text{m.s}^{-1} < v_x \leq 16.6 \text{m.s}^{-1} 
\end{cases} \]  

(22)

**Figure**: Switching signal \( \sigma(t) \).
Experimental validation

Solving the linear matrix inequalities in theorem 2, gives the solutions

\[
P_1^1 = \begin{bmatrix} 0.0080 & 0 \\ 0 & 0.0205 \end{bmatrix}, \quad P_2^1 = \begin{bmatrix} 0.0080 & 0 \\ 0 & 0.0205 \end{bmatrix}, \quad P_3^1 = \begin{bmatrix} 0.0080 & 0 \\ 0 & 0.3545 \end{bmatrix}
\]

\[
P_1^4 = \begin{bmatrix} 0.0080 & 0 \\ 0 & 0.3396 \end{bmatrix}, \quad P_2^1 = \begin{bmatrix} 0.2891 & 0 \\ 0 & 0.4252 \end{bmatrix}, \quad P_2^2 = \begin{bmatrix} 0.2891 & 0 \\ 0 & 0.4465 \end{bmatrix}
\]

\[
P_2^3 = \begin{bmatrix} 0.2870 & 0 \\ 0 & 0.5521 \end{bmatrix}, \quad P_2^4 = \begin{bmatrix} 0.2870 & 0 \\ 0 & 0.5521 \end{bmatrix}, \quad P_3^1 = \begin{bmatrix} 0.0673 & 0 \\ 0 & 0.3219 \end{bmatrix}
\]

\[
P_3^2 = \begin{bmatrix} 0.0673 & 0 \\ 0 & 0.3171 \end{bmatrix}, \quad P_3^3 = \begin{bmatrix} 0.0673 & 0 \\ 0 & 0.3407 \end{bmatrix}, \quad P_3^4 = \begin{bmatrix} 0.0673 & 0 \\ 0 & 0.3407 \end{bmatrix}
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Experimental validation

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\[ L_1^1 = 10^3 \begin{bmatrix} -0.0136 \\ 4.3830 \end{bmatrix}, \quad L_2^1 = 10^3 \begin{bmatrix} -0.0078 \\ 4.3813 \end{bmatrix}, \quad L_3^1 = \begin{bmatrix} -70.1966 \\ 260.5428 \end{bmatrix}, \quad L_4^1 = \begin{bmatrix} -68.2978 \\ 272.1352 \end{bmatrix} \]

\[ L_1^2 = \begin{bmatrix} -11.1729 \\ 153.0217 \end{bmatrix}, \quad L_2^2 = \begin{bmatrix} -6.2345 \\ 145.5492 \end{bmatrix}, \quad L_3^2 = \begin{bmatrix} -12.6483 \\ 120.0691 \end{bmatrix}, \quad L_4^2 = \begin{bmatrix} -7.7325 \\ 119.1450 \end{bmatrix} \]

\[ L_1^3 = \begin{bmatrix} -19.1574 \\ 331.7307 \end{bmatrix}, \quad L_2^3 = \begin{bmatrix} -13.6983 \\ 336.8248 \end{bmatrix}, \quad L_3^3 = \begin{bmatrix} -19.5678 \\ 314.5372 \end{bmatrix}, \quad L_4^3 = \begin{bmatrix} -14.0961 \\ 316.8263 \end{bmatrix} \]
Experimental validation

Figure: Interval observer of Lateral velocity.
Experimental validation

Figure: Interval observer of yaw rate.
Conclusion

- Robust estimation of lateral velocity and yaw rate using interval observers;

- Vehicle model subject to interval uncertainties (cornering stiffness & longitudinal velocity);

- The simulation results demonstrate the validity of the proposed approach.

- The convergence time is short and the intervals width are tight.
Thank you for your attention!

Robust estimation of vehicle lateral velocity and yaw rate using Switched T-S Fuzzy Interval Observer

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